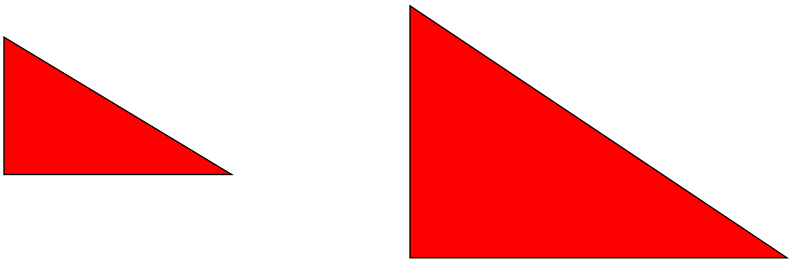
Lesson 5: Similar Triangles

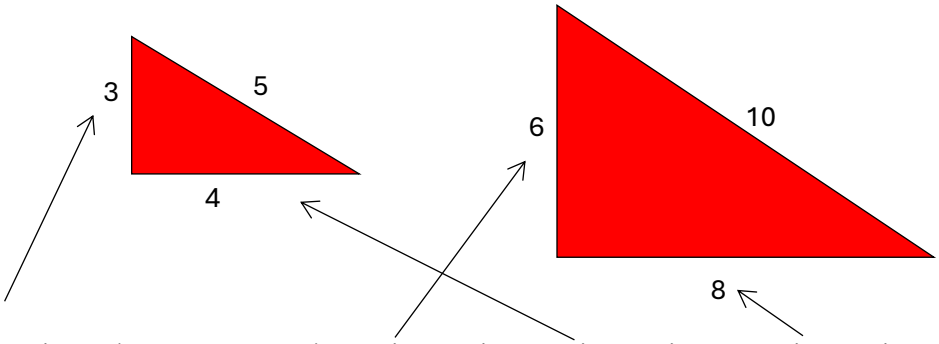
In the last lesson, you learned that congruent triangles are exactly the same. They are the same size and the same shape with the same angles. Now you will learn about *Similar Triangles*.

Similar Triangles are…well…similar, but not the same size. The two triangles below are the same shape with the same angles, but the sides are different lengths. That makes them similar, but not congruent.



I realize this sounds ridiculous and you are probably wondering why I would even bother telling you this, but there is a point, I promise. First, we will learn about proportions because they are fun and then we'll learn how to use similar triangles in story problems, which is not as fun.

I have given a length for each side of the two similar triangles below. Now we will apply our proportions skills to see something cool.



This side is (in proportion) to this side, as this side is to this side. I'll write that as a proportion.

$$\frac{3}{6} = \frac{4}{8}$$

This ratio means "3:6" (3 to 6) and this ratio means "4:8" (4 to 8). When you have "this ratio equals that ratio" you have a proportion. To prove that these two are equal, we can cross multiply. If the answer is the same in both directions, then they are equal.

$$3 \times 8 = \mathbf{24} \qquad 6 \times 4 = \mathbf{24}$$

$$\frac{3}{6} = \frac{4}{8}$$

Yep, they are equal. Now, let's suppose we didn't know one of those numbers. I will replace the 4 above with an x.

$$\frac{3}{6} = \frac{x}{8}$$

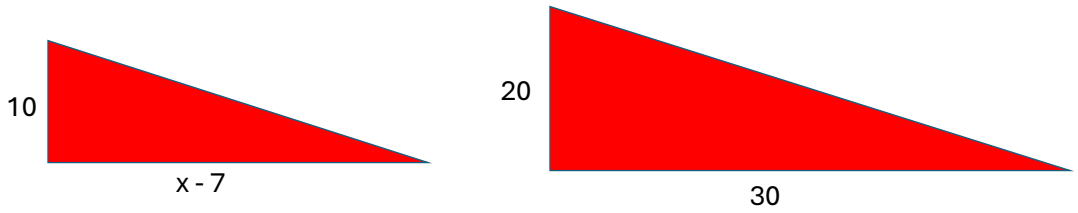
Now I'll cross multiply again, $3 \times 8 = \mathbf{24}$ *and* $6 \cdot x = \mathbf{6}x$.

$$24 = 6x$$

Solve for x.

$$4 = x$$

Tada! OK, that was easy. Take a look at these next two similar triangles.



Let's do the same thing. I've written the proportions below.

$$\frac{10}{20} = \frac{x - 7}{30}$$

Again, we cross multiply and solve for x.

$$10 \cdot 30 = 300 \quad and \quad 20(x-7) = 20x - 140$$

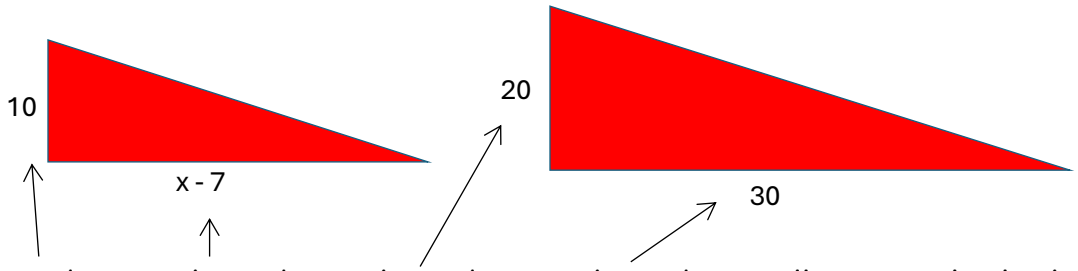
$$300 = 20x - 140$$

$$300 + 140 = 20x$$

$$440 = 20x$$

$$22 = x$$

If x = 22, then the base of that last triangle is 15. But watch this! I'll bring back that last problem. It is also true to say that…



This side is to this side, as this side is to this side. I will cross multiply those ratios and then watch; I will get the same answer.

$$\frac{10}{x-7} = \frac{20}{30}$$

$$300 = 20x - 140$$

$$440 = 20x$$

$$x = 22$$

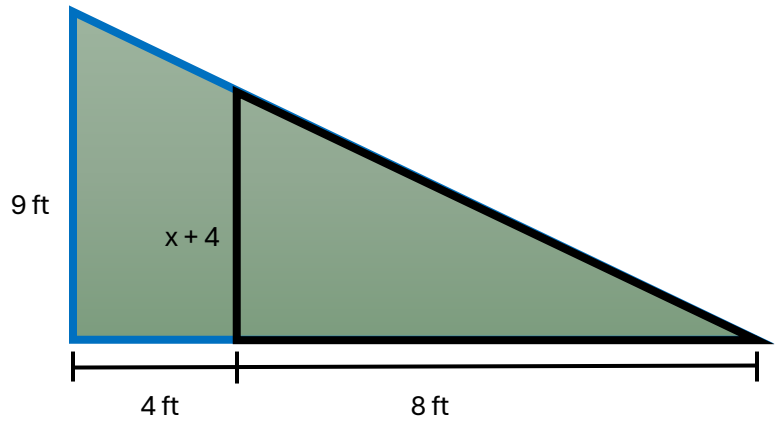
The trick is to make sure each ratio is built the same way. In the first example, I was sticking to this formula:

$$\frac{Little\ Triangle's\ height}{Big\ Triangle's\ height} = \frac{Little\ Triangle's\ Base}{Big\ Triangle's\ Base}$$

The second time I solved the same example I used this formula:

$$\frac{Little\ Triangle's\ Height}{Little\ Triangle's\ Base} = \frac{Big\ Triangle's\ Height}{Big\ Triangle's\ Base}$$

Make sure your ratios are consistent and logical. Next, I am going to overlap two triangles. Do you see the two similar triangles below? The big triangle is outlined in blue, and the smaller triangle is outlined in black.



We are going to solve for x in the triangle above. It is solved the same way as before; just make sure the base of the big triangle is 12 ft.

Height → $\frac{9}{x+4} = \frac{12}{8}$ ← Base

Do you know what to do now? That's right, cross multiply.

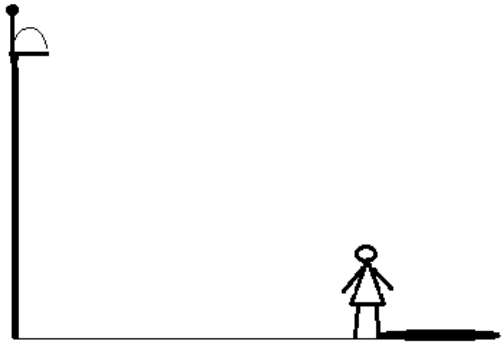
$$\frac{9}{x+4} = \frac{12}{8}$$

$$72 = 12x + 48$$
$$72 - 48 = 12x$$
$$24 = 12x$$

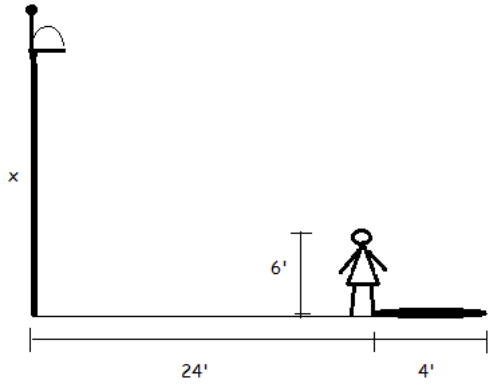
$$2 = x$$

Now let's apply this knowledge to a story problem. Here is a classic story problem you might find on a math test.

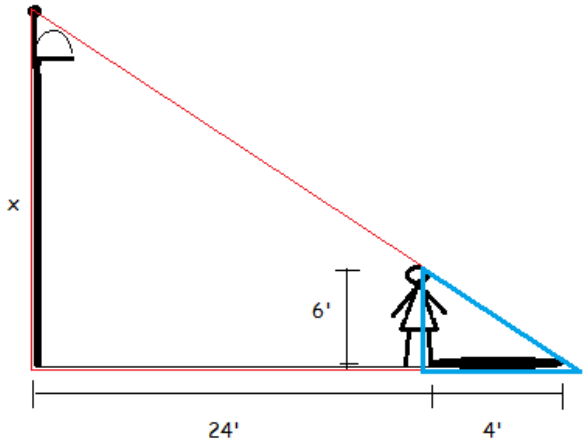
A 6-foot-tall man is standing 24 feet from a light post. The length of his shadow created by the light is four feet. Find the height of the streetlight.



Do you know how to solve this problem? Believe it or not this is a Similar Triangle problem. To help us understand the question, I have added the measurements to the picture below.



Next, I will draw the two similar triangles over top of the picture.



Now we can create an equation to help us solve for x. The larger triangle is "x feet" tall and the man is 6 feet tall, so the first ratio is $\frac{x}{6}$. The base of the big triangle is 28' and the smaller triangle's is 4'.

$$\frac{x}{6} = \frac{28}{4}$$

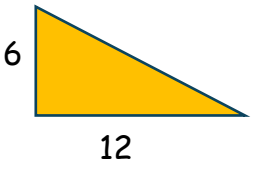
I will cross multiply to solve for x.

$$4x = 168$$

$$x = 42\ feet$$

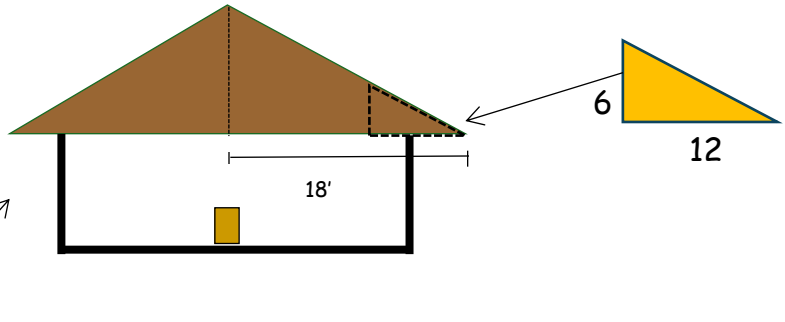
The light post is 42 feet tall.

Home builders use similar triangles to describe how steep a roof is or will be. Just like *slope* measures how steep a line is, builders use "pitch" to show how steep a roof is to be built. For example, let's say the pitch of a roof is 6:12. That means that for every 12 horizontal inches (or 12 feet), the roof will go up by 6 vertical inches (or feet).



This shape is similar to the pitch of a roof, just much smaller. We can use it proportionately to find some unknown dimensions. Look at the brown roof on the

house in the drawing. Half of the triangular rooftop is similar to our "6:12 pitch" triangle. So, if I know the length of the base or the height of the triangle, I could figure out the 3rd dimension.



If the base of this brown triangle measures 36 feet, then half of it is 18 feet. Here are the ratios of our two triangles.

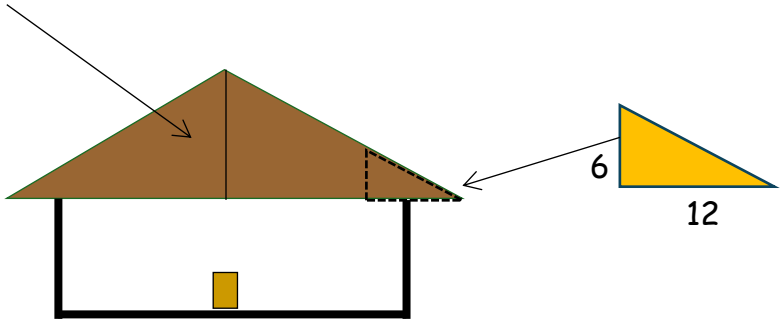
Small triangle → $\frac{6}{12} = \frac{x}{18}$ ← Actual roof

Cross multiply to solve for x.

$$12x = 108$$

$$x = 9$$

The height is 9 feet.



Just make sure that you are always consistent when creating proportions! By that I mean, the numbers on top must come from the same place. For example, let's say the best ratio of ice cubes:pop is four ice cubes per 12 ounces of pop. Less ice wouldn't make it cold enough and more ice would make the cup overflow. That ratio would be written like this:

$$\frac{4\ ice\ cubes}{12\ ounces\ of\ pop}$$

Then, if you wanted to find the amount of ice cubes needed for 48 ounces of pop, you would write this next ratio. JUST MAKE SURE THE TWO RATIOS BOTH HAVE ICE CUBES IN THE SAME SPOT.

$$\frac{4\ ice\ cubes}{12\ ounces\ of\ pop} = \frac{x\ ice\ cubes}{48\ ounces\ of\ pop}$$

It's OK if you switch around the ice cubes and the ounces, you'll get the same answer. Just make sure you stay consistent!

$$\frac{12\ ounces\ of\ pop}{4\ ice\ cubes} = \frac{ounces\ of\ pop}{x\ ice\ cubes}$$

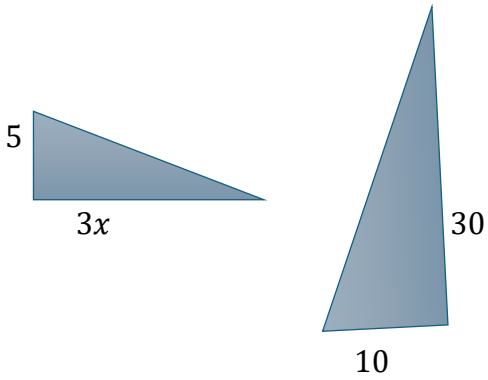
Complete the next worksheet.

Worksheet 5

Name _____ Date _____

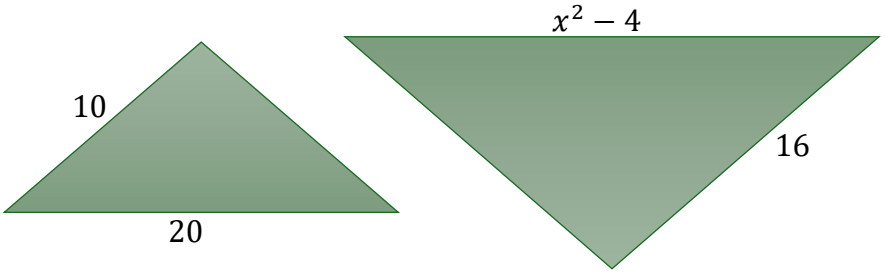
Solve for "x" in the following sets of similar Triangles.

1.



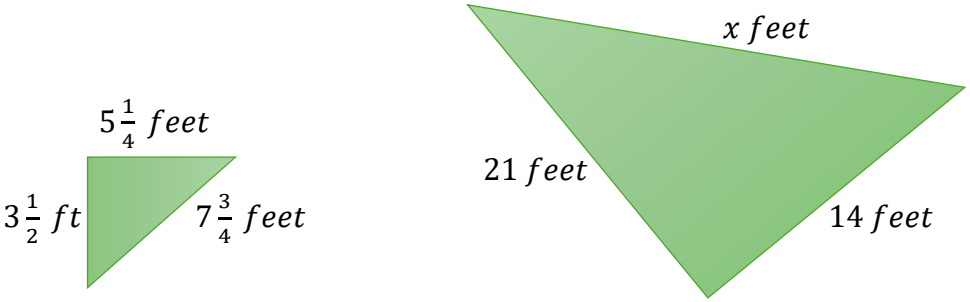
_____

2.



_____

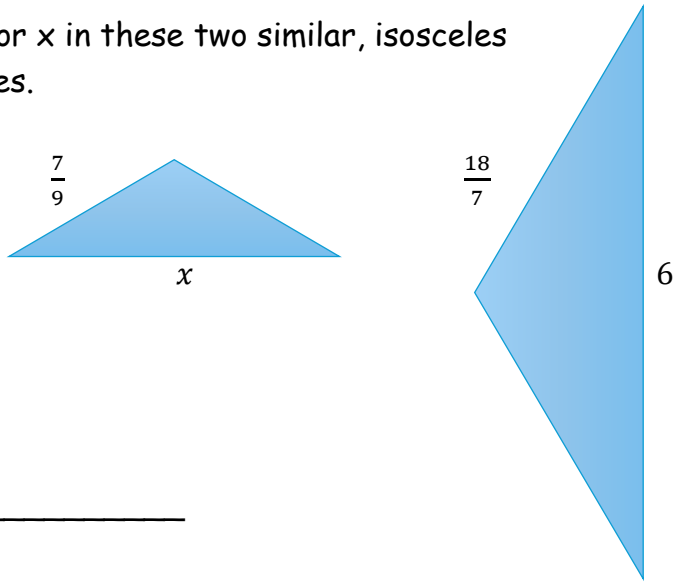
3.



_____

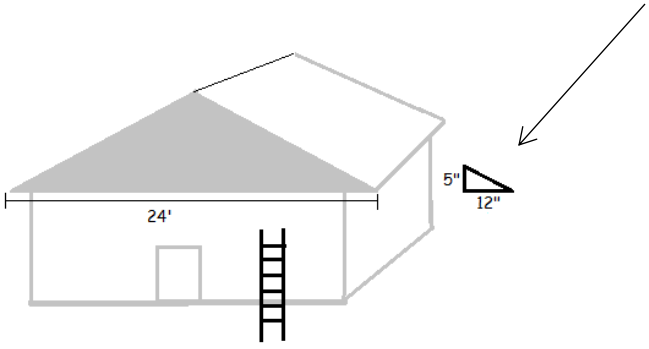
Worksheet 5 page 2

4. Solve for x in these two similar, isosceles Triangles.



_____

5. A builder wants to know the square footage of the shaded triangular area of the building below. His ladder isn't tall enough to measure the height of the triangle, so he will have to use math instead. He knows that the pitch of the roof is 5:12. Use similar triangles to find the area of the shaded area below. (HINT: Slice the gray triangle in half, so it is similar to the pitch).



_____