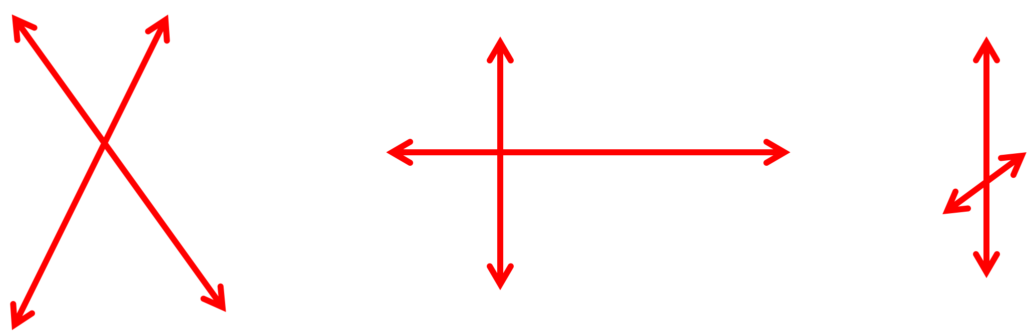
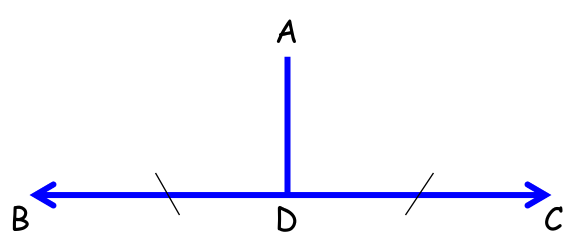
Lesson 8: Altitude, Medians, and Bisectors

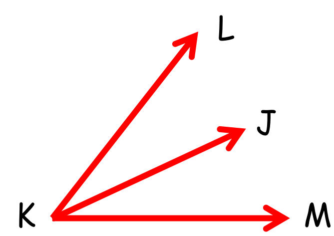
Have you ever been stopped at a stop light? That traffic light was more than likely at an intersection. When two roads cross paths, they form an intersection. In geometry, when two lines cross paths, we say this line "intersects" that line.
Here are some examples of intersecting lines.



Nothing special here; just two lines crossing or intersecting each other. There are no promises made about the angle sizes or the length of the lines. Intersecting lines just cross each other. A BISECTOR is another story. A *bisector* is a line that cuts another line into two equal line segments. For example, line AD below bisects line BC. That makes $\overline{BD} \cong \overline{DC}$. No need to measure the two lines. Since I said that line AD is a BISECTOR, the two line segments are PROVEN to be congruent.



A bisector can also cut an angle into two equal angles.

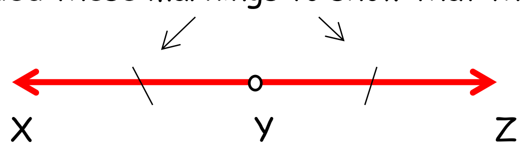


$\overrightarrow{JK}$ bisects ∠ LKM.
$\angle LKJ \cong \angle JKM$

A bisector that cuts an angle into two congruent angles is called an Angle Bisector. Those were easy ones. Next you will learn the definition of a *midpoint*; it's an easy one too. If you take the words "middle" and "point" and squish them together you get midpoint.

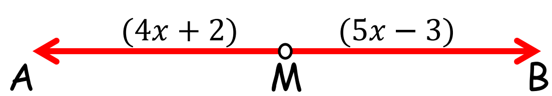
MIDDLE + POINT = MIDPOINT

Point Y is the midpoint of line XZ. That makes line segment XY congruent to line segment YZ. I added these markings to show that they are equal lengths.



Try this one.

Given: M is the midpoint of line AB.
Solve for x.



Do you know how to solve this one? Since point M is the MIDPOINT, line segment AM is congruent to line segment MB, so the two lengths are equal. I'll write that out in an algebraic equation and then solve for x.

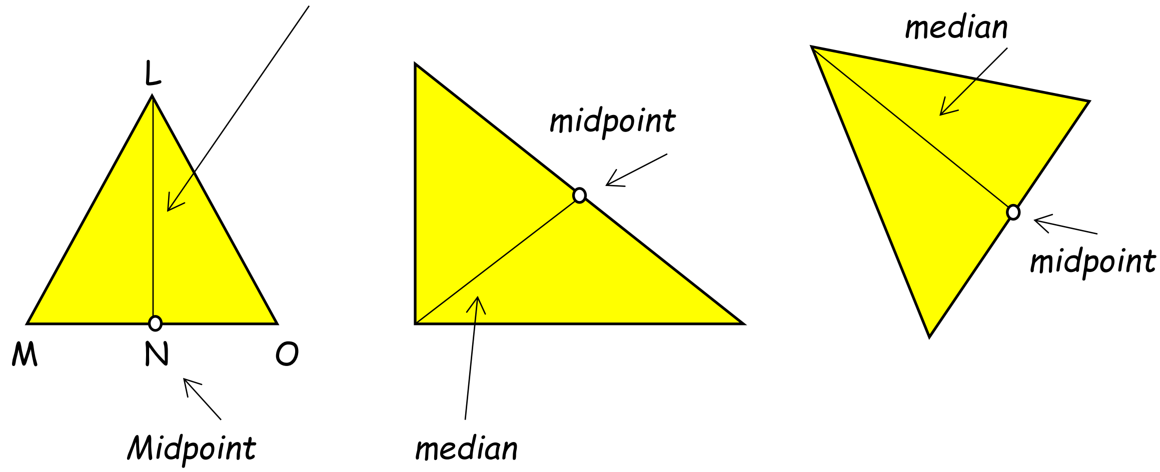
$$4x + 2 = 5x - 3$$

$$2 + 3 = 5x - 4x$$

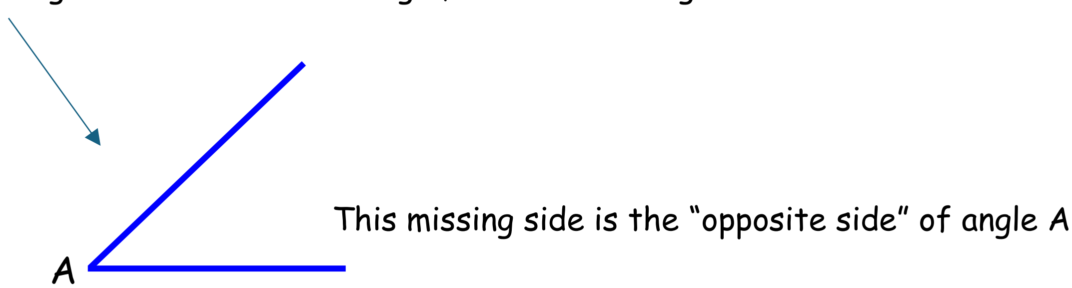
$$5 = x$$

Do you see how a bisector and midpoint are similar? A bisector cuts a line or angle into two equal parts. A midpoint is merely the point that shows the center of a line segment.

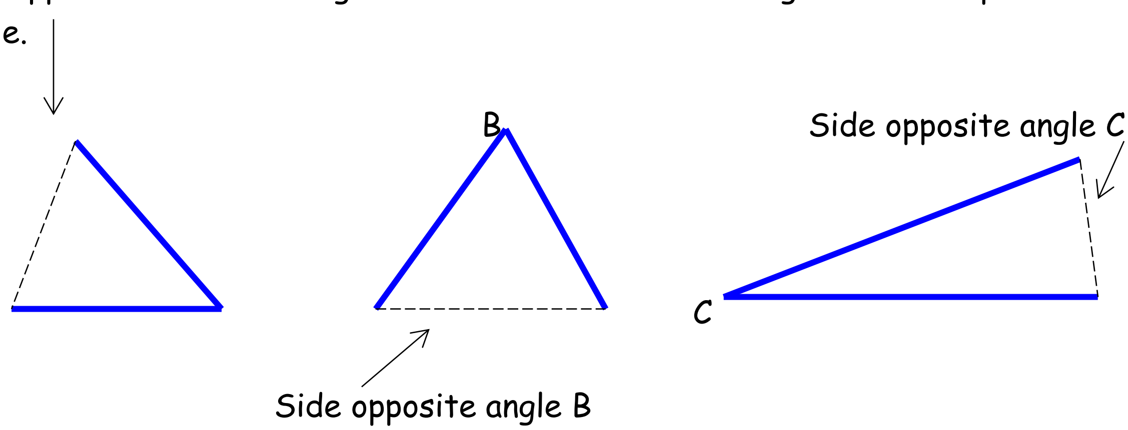
Next you will learn how the *median of a triangle* is similar as well. Take any triangle and draw a line from any vertex to the midpoint of the opposite side. You have just drawn the median of the triangle.



Let me explain what I mean when I say the "side opposite the angle." Look at this next angle. It would be a triangle, but it is missing a side.



The opposite side of an angle is the third side of a triangle that isn't part of that angle.

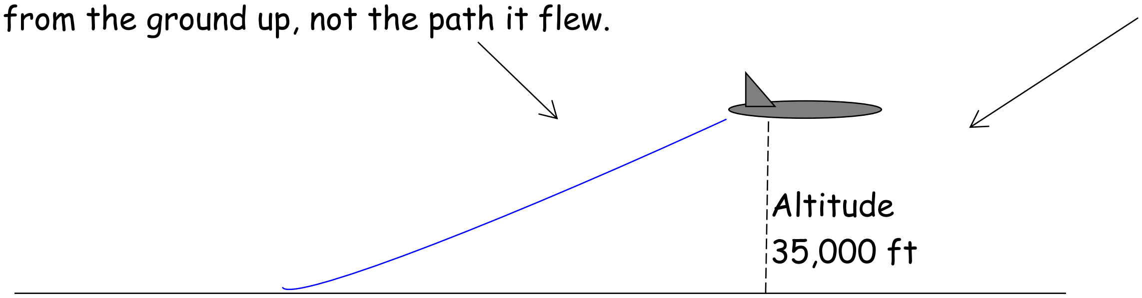


It kind of reminds me of a flashlight's beam of light. Picture those angles as the light coming from a flashlight and hitting a wall. The "wall" that the beam of light hits is the *opposite* side of the angle. This may seem silly or ridiculous, but later in this book you will REALLY need to know which side is opposite an angle, so let's start learning it now.

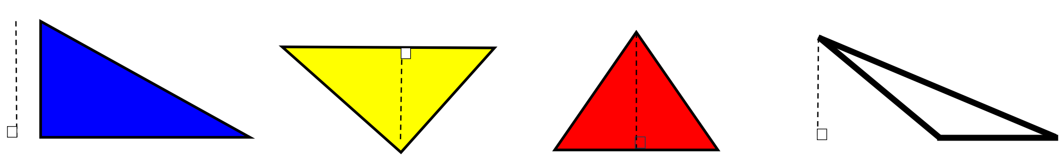
Next, we will learn a term that is just as easy as the last three, but it has nothing to do with the middle or half. This term is all about height.

Do you have any mountains near your hometown? I can see Mt. Rainier from my house. I'm told that Mt. Rainier has an altitude of 14,409 feet. To measure a mountain, you measure from the ground to the top of the mountain. You wouldn't measure the side.

Have you been in an airplane? An airplane starts on the ground and then flies up to an altitude of about 35,000 feet. To measure the altitude of a plane you measure from the ground up, not the path it flew.

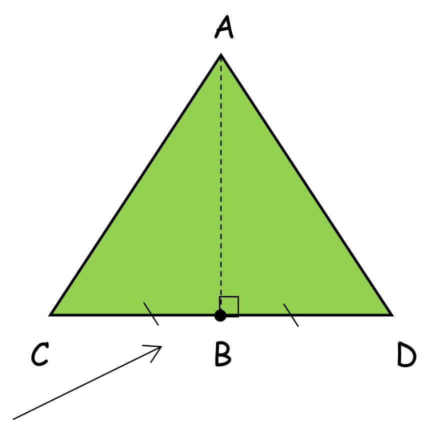


The *altitude* of a triangle is measured the same way. You measure from the ground up. It is always a vertical line that is perpendicular to the base of the triangle. Here are four different examples of triangles with their altitudes in dotted lines.



Let's recap the four new terms we just learned and then I'll show you how these terms can help us find missing values.

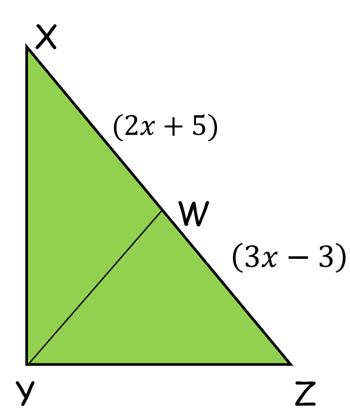
Look at the equilateral triangle below. Line AB is a **bisector** because it splits line CD into two equal line segments. It is also an **angle bisector** because it splits angle A into two equal (congruent) angles. Line AB is also the **median** of the triangle because it starts at a vertex and extends to the **midpoint** of the **opposite side**. And $\overline{AB}$ marks the **altitude** of triangle ACD too, because it is a vertical line drawn from the tallest part of the triangle to the base and is perpendicular to the base.



Point B is the **midpoint** of line CD because line segment CB is congruent to line segment BD.

So now let's put our new knowledge to work. Look at the shape below. Read the given clues and then solve for x.

Given: YW is the median to XZ
Solve for x.

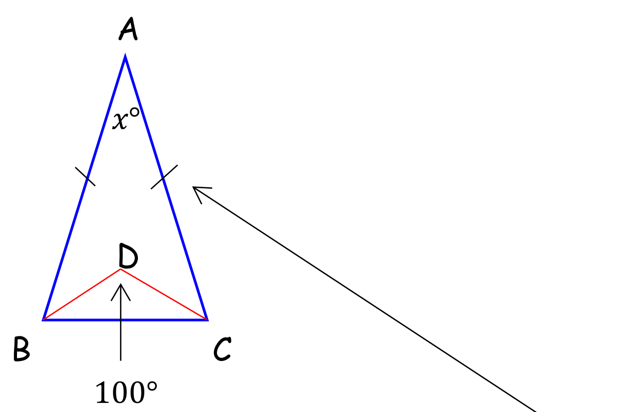


Do you know what to do? Since we know that $\overline{YW}$ is the median, we know point W is the midpoint. So, line segment XW has to be congruent to line segment WZ. Let's write that out in an algebraic equation and then solve for x.

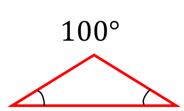
$$(2x + 5) = (3x - 3)$$
$$5 + 3 = 3x - 2x$$
$$8 = x$$

Do you get how this works? Since we know the meaning of the word "median" and we know how to do algebra, we can combine these skills to solve geometry. Let's try another one. Read the given clues and then solve for x.

Given: $\overline{BD}$ and $\overline{CD}$ are angle bisectors.



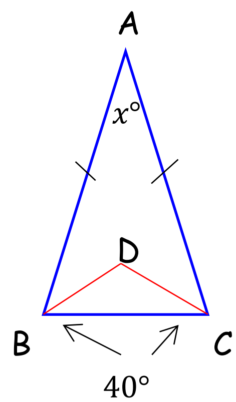
This one is a little more challenging. Can you solve for x? Did you notice these little marks? Those marks mean that the two sides are congruent. What does that mean? It means we are dealing with an isosceles triangle. What does that mean? It means that angles B and C are also congruent. And what does an angle bisector do? It splits an angle into two congruent angles. So, if angles B and C are equal and we split both of them in half, are the two "half angles" equal? Yes, yes they are because ***halves of equals are equal.*** Let's take a look at just the little triangle for now.



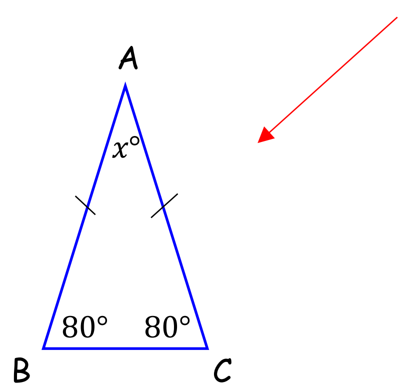
Can you figure out the measure of those two base angles? Well, if the vertex angle is $100°$ and the other two angles are congruent, let's do some math.
We know that all three angles must equal $180°$, so let's subtract the vertex angle and see how much is left.

$$180° - 100° = 80°$$

We know that the two base angles are the same, so let's split up the remaining $80°$ into two equal angles. Now we know all three measurements to the little triangle. Can you solve for x now?



If each one of the "half angles" are $40°$, then the full-sized angles B and C must be $80°$ each. So now let's focus our attention back onto the bigger triangle ABC.



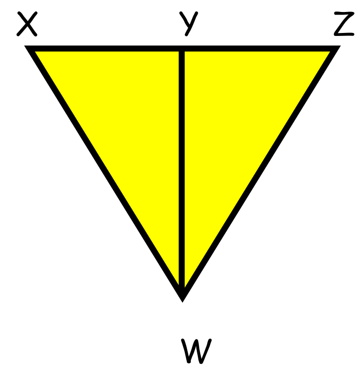
Again, all three angles must equal $180°$, so let's do the math.

$$80 + 80 + x = 180$$
$$160 + x = 180$$
$$x = 180 - 160$$
$$x = 20°$$

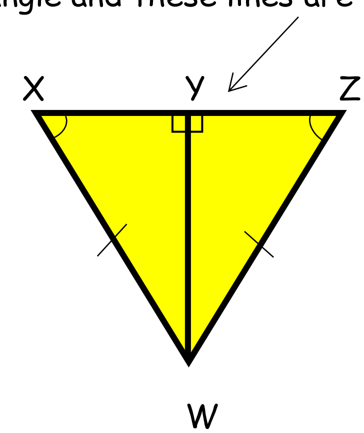
Let's try one more together, but this time we'll use some of the stuff we learned from an earlier lesson too. Look at the next drawing, read the given clues, and then prove that Y is the midpoint of side XZ.

Given: $\overline{WY}$ is the altitude of $\Delta\, XWZ$.
$\overline{WX} \cong \overline{WZ}$.

Prove: Y is the midpoint of XZ.



So, let's think about this. If Y **IS** the midpoint, then XY would have to be congruent to YZ. That's what we need to prove. How can we do that? Well let's look at what we know for sure and then mark up our drawing with that information. We are told that WY is the altitude. What does that mean? It means two things; it is the height of the triangle and these lines are perpendicular to each other.



We also know that the two sides are equal, so this must be an isosceles triangle. That means that angles X and Z are congruent. So, wait a minute. Are you telling me that we have two right triangles, back-to-back? If we can prove that these two right triangles are congruent, then we can prove that XY and YZ are congruent too. Because remember, earlier we learned that Corresponding Parts of Congruent Triangles are Congruent, or CPCTC for short.

So, let's see, these are right triangles, so let's try to prove they are congruent with the Hy-Leg Postulate. Are the hypotenuses congruent? Yes, they are. How about a leg, do they have at least one congruent leg? Well yeah, they are sharing a leg, so it is obviously congruent to itself (Reflexive Property). Well, there you go. We just proved the two right triangles are congruent by the Hy-Leg Postulate. And Corresponding Parts of Congruent Triangles are Congruent (CPCTC), so we just proved that XY is congruent to YZ, therefore, Y is indeed the midpoint.

Wow, that was intense. Try some on your own by completing the next worksheet. If you have problems, take a little peek at the answers to help steer you in the right direction. If you have to peek at the answers for each problem, then you should read this lesson again and then try the worksheet again.

Name ____ Date ___________

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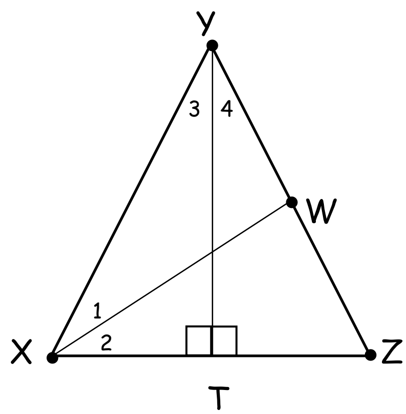
Questions 1 – 10 are about this drawing.



1. $\overline{XW}$ is the bisector of $\overline{YZ}$. If $\overline{YW}$ = 5, then $\overline{WZ}$ = _____.

2. If $\triangle$XYZ is isosceles, $\overline{XY}$ = 10, and point W is the midpoint of $\overline{YZ}$, then what is the measure of line segment $\overline{WZ}$? _______

3. Name the altitude of $\triangle$XYZ. ________

4. $\measuredangle 1 = 35°$, $\overline{XW}$ is an angle bisector. What is the measurement of $\angle 2$? ______

5. $\overline{XW}$ is the median and $\overline{YZ}$ = 12. What is the measurement of $\overline{YW}$? _______

6. YT is the altitude of $\triangle$XYZ. If $\measuredangle 4 = 32°$, then $\measuredangle Z =$ ______

7. Point T is the midpoint of $\overline{XZ}$. Which Postulate proves $\triangle XYT \cong \triangle ZYT$?

   ____

8. $\overline{XW}$ is an angle bisector in isosceles $\triangle$XYZ. If $\measuredangle Z = 50$, then $\measuredangle 2 =$ ______.

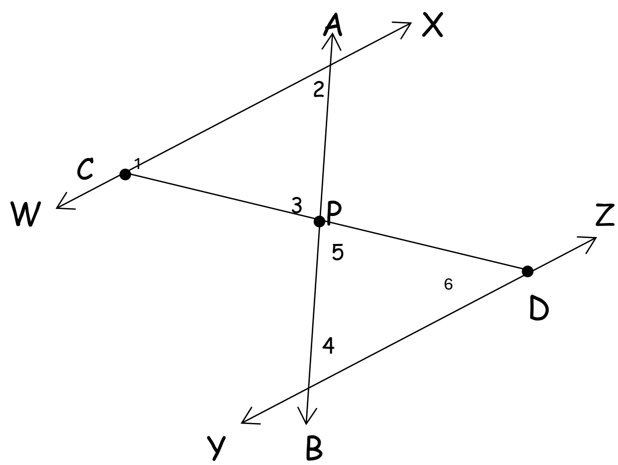
Worksheet 8 page 2 of 2

9. Point T is the midpoint of XZ. YT is the altitude and the _________ of △XYZ.

10. YT is an angle bisector in isosceles △XYZ. If $\measuredangle X = 70$, then $\measuredangle 3 =$ ____

11. Given: $WX \parallel YZ$, AB bisects CD at point P

Prove: $\triangle APC \cong \triangle BPD$



STATEMENT	REASON
1. $WX \parallel YZ$	1.
2. $\measuredangle 1 \cong \measuredangle 6$	2.
3. $\measuredangle 3 \cong \measuredangle 5$	3.
4. $CP = PD$	4.
5. $\triangle APC \cong \triangle BPD$	5.